

$$\csc^2 A + \sec^2 A = (\cot A + \tan A)^2$$

↓

$$= \cot^2 A + 2\cot A \tan A + \tan^2 A$$

$$= \cancel{\csc^2 A - 1} + 2 \frac{1}{\cancel{\tan A}} \cancel{\tan A} + \cancel{\sec^2 A - 1}$$

$$= \csc^2 A + \sec^2 A$$

$$\frac{\cos x}{1 - \sin x} - \frac{1 - \sin x}{\cos x} = 2 \tan x$$

↓

$$= \frac{\cos^2 x - (1 - \sin x)^2}{(1 - \sin x) \cos x}$$

$$= \frac{\cos^2 x - (1 - 2 \sin x + \sin^2 x)}{(1 - \sin x) \cos x}$$

$$= \frac{\cancel{1} - \sin^2 x - \cancel{1} + 2 \sin x - \sin^2 x}{(1 - \sin x) \cos x}$$

$$= \frac{2 \sin x - 2 \sin^2 x}{(1 - \sin x) \cos x}$$

$$= \frac{2 \sin x (1 - \sin x)}{(1 - \sin x) \cos x}$$

$$= 2 \tan x$$

$$\frac{1}{(1 - \sec^2 \beta) \cos \beta}$$

$$= \frac{1}{-\tan^2 \beta \cos \beta}$$

$$= \frac{1}{-\frac{\sin^2 \beta}{\cos^2 \beta} \cos \beta}$$

$$= -\frac{\cos \beta}{\sin^2 \beta}$$

$$= -\frac{\cos \beta}{\sin \beta} \cdot \frac{1}{\sin \beta}$$

$$= -\cot \beta \csc \beta$$

$$\frac{\cos \theta}{\tan \theta + \sec \theta}$$

$$= \frac{\cos \theta}{\frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}}$$

$$\frac{\cos \theta}{\cos \theta}$$

$$= \frac{\cos^2 \theta}{\sin \theta + 1}$$

$$= \frac{1 - \sin^2 \theta}{\sin \theta + 1}$$

$$= \frac{(1 - \sin \theta)(1 + \sin \theta)}{\cancel{\sin \theta + 1}}$$

$$= 1 - \sin \theta$$

Prove $\frac{\cot C - \tan C}{\cos C + \sin C} = \frac{\cos C - \sin C}{\cos C \sin C}$



$$= \frac{\frac{\cos C}{\sin C} - \frac{\sin C}{\cos C}}{\cos C + \sin C} \cdot \frac{\sin C \cos C}{\sin C \cos C}$$

$$= \frac{\cos^2 C - \sin^2 C}{(\cos C + \sin C) \sin C \cos C}$$

$$= \frac{(\cos C - \sin C) \cancel{(\cos C + \sin C)}}{(\cos C + \cancel{\sin C}) \sin C \cos C}$$

$$= \frac{\cos C - \sin C}{\sin C \cos C}$$